



You should get the following results:

X	Y1
.1	2.5937
.01	2.7048
.001	2.7169
1E -4	2.7181
1E -5	2.7183
1E -6	2.7183

X=

Do you recognize this number?

This table is analogous to the one-sided limit as  $x$  approaches zero from the right:

$$\lim_{x \rightarrow 0^+} (1 + x)^{\frac{1}{x}}$$

We now need to investigate what the limit of this function is as  $x$  approaches zero from the left:

$$\lim_{x \rightarrow 0^-} (1 + x)^{\frac{1}{x}}$$

To do this we follow step 3 but now we input the values: -.1, -.01, -.001, -.0001, -.00001 and -.000001.

See next page for the corresponding  $y$ -value for these  $x$ -values and the output of the new table.

X	Y1
-.1	2.868
-.01	2.732
-.001	2.7196
-1E -4	2.7184
-1E -5	2.7183
-1E -6	2.7183

X=

Therefore we can conclude the following:

Since  $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = \lim_{x \rightarrow 0^-} (1+x)^{\frac{1}{x}} = 2.7183$  then  $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = 2.7183$ .

So what is this number 2.7183 you may ask?

Step 4. You should press 2<sup>nd</sup> MODE to quit the TABLE view. Now we are going to find the value of Euler's Number  $e$ . Find the LN key on your calculator and above it you will see  $e^x$ . Hence you should type  $e^{(1)}$  ENTER and the answer is 2.718281828 that when rounded to the nearest ten thousandths it's the value we found in the table.

## On your own (Blog Activity)

Using the steps above (do not delete  $y_1$ ) and define a new function:  $y_2 = \left(1 + \frac{1}{x}\right)^x$ .

What can you conclude about the following limit?

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

Before you begin the investigations, in the blog please discuss the following:

- What do you expect to happen? Explain your reasoning.
- Do you see any similarities between the functions  $y_1 = (1+x)^{\frac{1}{x}}$  and

$$y_2 = \left(1 + \frac{1}{x}\right)^x ?$$

Now use your calculator to evaluate the limit numerically.

(HINT: Use 99, 999, 9999, 99999 and 999999 as approximations of infinity ( $\infty$ ))

## Extension (Hand-In)

Clear all functions from above. Use your calculators to find a pattern to the following limits. Describe the pattern and write a general rule for evaluating such limits.

a.  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$

b.  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x =$

c.  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x =$

d.  $\lim_{x \rightarrow \infty} \left(1 + \frac{\pi}{x}\right)^x =$

PATTERN:

RULE: